Use counter-examples to disprove a conjecture

1 Complete the sentences using the correct words from the list.
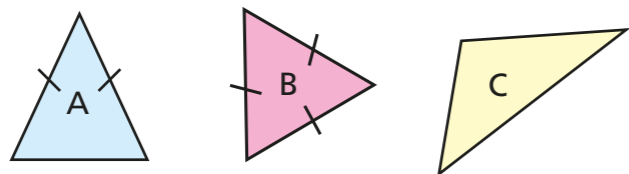
| true | false | one | ten | five |

A counter-example is an example that shows a conjecture to be ___false___

___One___ counter-example is needed to disprove a conjecture.

2 Which diagram is a counter-example for the statement?

> A triangle always has at least one line of symmetry.

A   B   C

Triangle ___C___ is a counter-example for the statement.

3 Which of the values of x and y is **not** a counter-example for the statement? Circle the correct answer.

> For any numbers x and y, $\quad x - y = \dfrac{x}{y}$

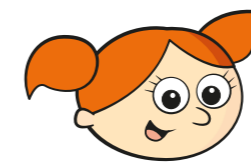$x = 6$ and $y = 3$ $x = 8$ and $y = 4$

(x = 4 and y = 2) $x = 10$ and $y = 4$

Explain to a partner how you know.

4

> The factors of 8 are 1, 8, 2 and 4

> So it must be true that all numbers have an even number of factors, as they can be written in pairs.

a) Write a counter-example to show that this conjecture is not always true.

25 has three factors 1, 5 and 25

b) What type of number has an odd number of factors?

___Square___

5 Tommy is adding numbers together.

> The sum of two numbers is always greater than each of the numbers.

Amir says Tommy is incorrect.

His counter-example to Tommy's conjecture is $\dfrac{1}{2} + \dfrac{1}{3}$

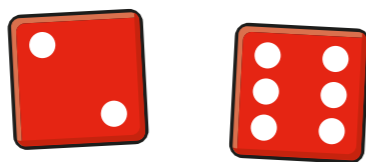Do you agree with Amir and his example?

No. $\dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}$ and $\dfrac{5}{6} > \dfrac{1}{2}$ and $\dfrac{5}{6} > \dfrac{1}{3}$

Find a different counter-example to Tommy's conjecture.

$3 + -2 = -1$ $-1 < 3$

6 Teddy rolls two dice.

a) He finds the product of the scores and makes a conjecture.

> The product of the scores on the two dice is always even.

Find a counter-example.
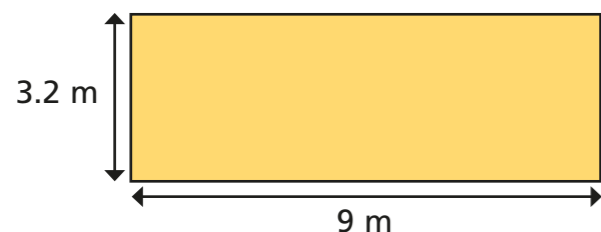
_____ $1 \times 5 = 5$ _____

b) He conjectures again.

> If the product of the scores on the two dice is even, then the sum of the scores will also be even.

Is this conjecture correct? If not, find a counter-example.

_____ No. $1 \times 2 = 2$ but $1 + 2 = 3$ _____

7 Dora works out the perimeter and area of this rectangle

3.2 m

9 m

perimeter = 24.4 m

area = 28.8 m²

Dora makes this conjecture.

> If a rectangle's perimeter is not an integer, then neither is its area.

Do you agree with Dora's conjecture? _No_

Justify your answer.

_____ $3.25 + 3.25 + 8 + 8 = 22.5$ _____

_____ $3.25 \times 8 = 26$ _____
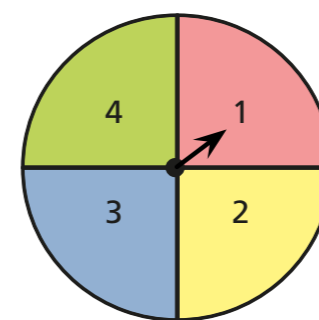
8 The range of five numbers is 1

The smallest number is 3

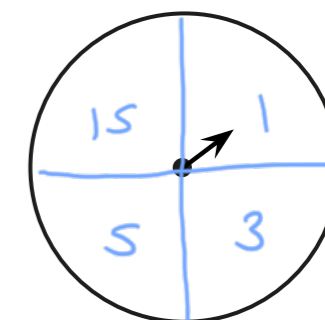Aisha thinks the median must be 3.5, as the greatest number is 4 and 3.5 is in the middle.

Give a counter-example to Aisha's conjecture.

_____ e.g.   3, 3, 3, 4 _____

9 Two spinners are spun and the results are multiplied.

spinner 1

spinner 2

The second spinner has the factors of 15 written on it.

Huan makes a conjecture about the product of the two numbers.

Esther finds a counter-example, which is 1 × 1 = 1

What was Huan's conjecture?

_____ e.g. The product is always odd. _____

Is there more than one possible conjecture?